Sound Resonance and Damping in Cylindrical Cavities

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Rebecca Barry, Dr. Erich Gust

Introduction

We are looking at sound resonating in a cylindrical cavity for different gases by measuring their resonant frequencies and their widths. In principle, the widths of the peaks are related to the microscopic properties of the gas molecules. This relation occurs because the width of the resonance is proportional to the damping constant. The damping of sound waves depends on viscosity, thermal conductivity, and the fine details of the cavity. Both viscosity and thermal conductivity occur because of molecular collisions.

Theory

The Helmholtz equation can describe sound waves. It must be solved in cylindrical symmetry with the boundary condition of 'zero normal derivative':

$$k^2\phi + \nabla^2\phi = 0$$

The quantity, ϕ, is the velocity potential and k is the wave number. The velocity field of gas is the negative gradient of ϕ:

$$\mathbf{v} = -\nabla\phi$$

The solutions to ϕ in cylindrical coordinates are:

$$\phi_{n,m,l}(r,\theta,z) = \cos\left(\frac{n\pi z}{L}\right) J_m\left(\frac{X_{m,l}r}{a}\right)\cos(m\theta)$$

The eigenvalues for k corresponding to these modes are:

$$k^2 = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{X_{m,l}}{a}\right)^2$$

Since frequency, $f = \frac{c}{\lambda} = \frac{ck}{2\pi}$, we have:

$$f_{n,m,l} = c\sqrt{\left(\frac{n}{2L}\right)^2 + \left(\frac{X_{m,l}}{2\pi a}\right)^2}$$

The presence of resonances causes the amplitude of the sound in the cavity to vary with frequency according to a Lorentzian peak shape:

$$V_n = \frac{F_n}{\Omega - f_n^{\circ} - i\Gamma_n}$$

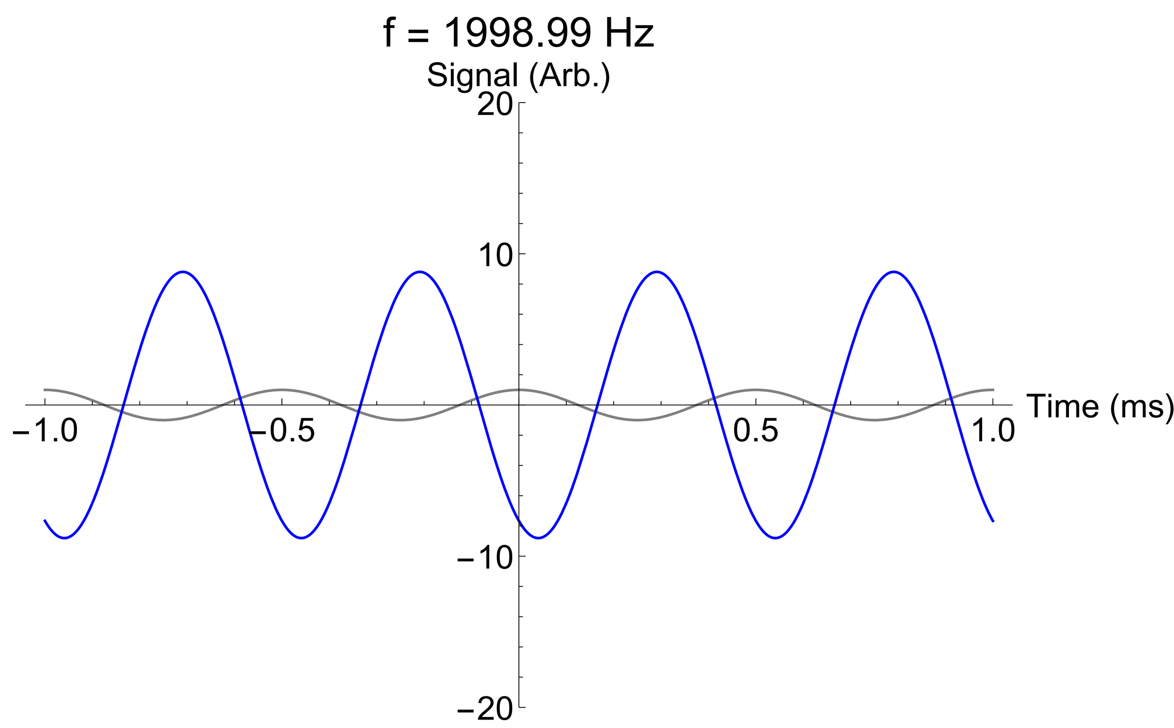

Figure 1: Signal vs. Time (ms). The gray signal represents the reference from the function generator, and the blue signal represents the signal from the microphone.

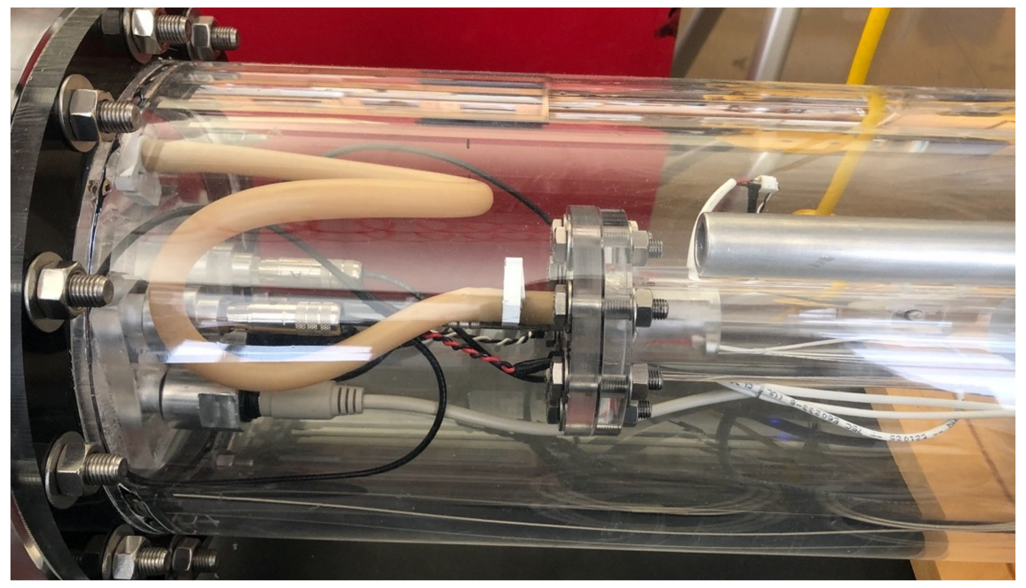

Figure 2: The cylindrical cavity

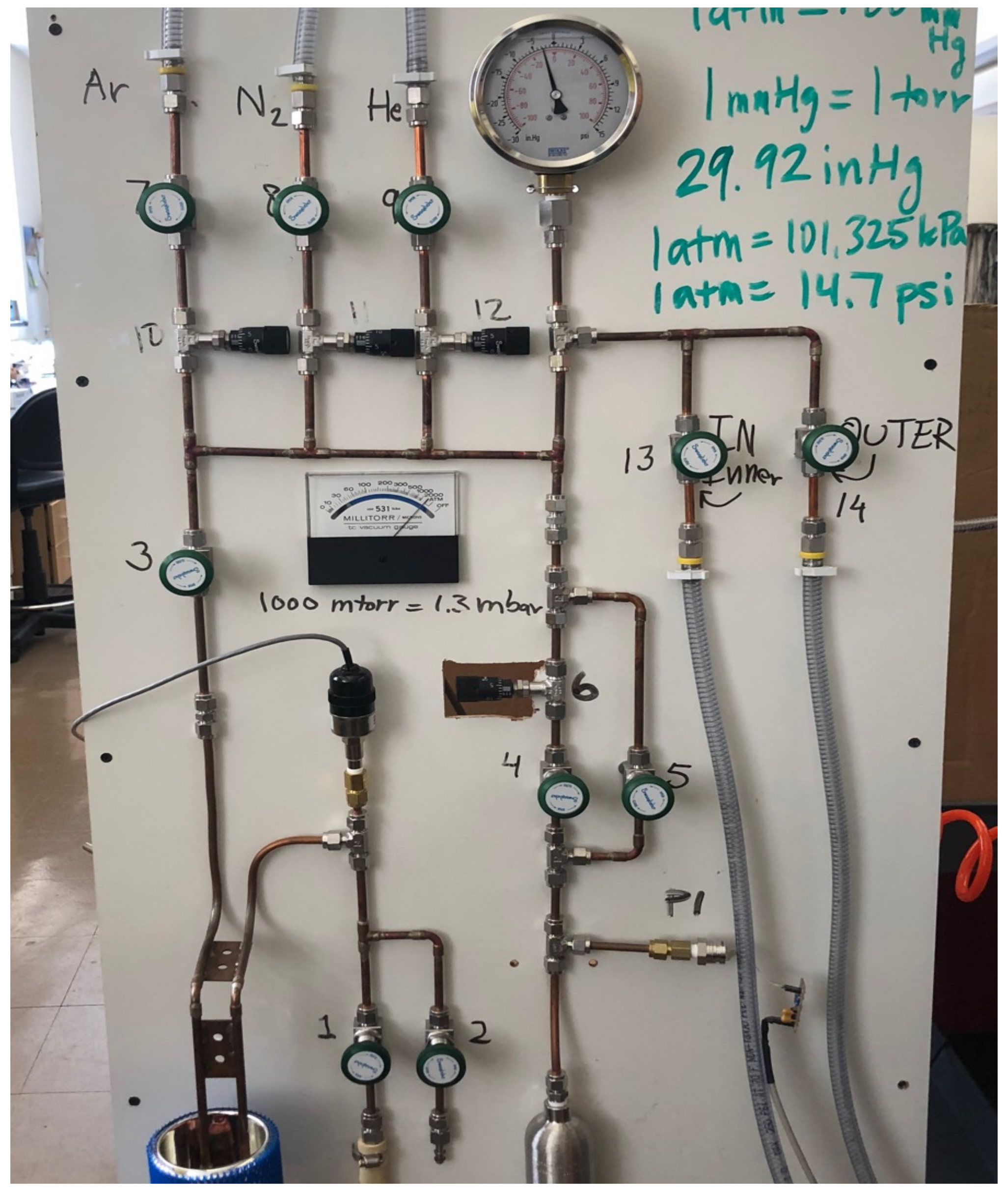

Figure 3: Gas-handling apparatus.

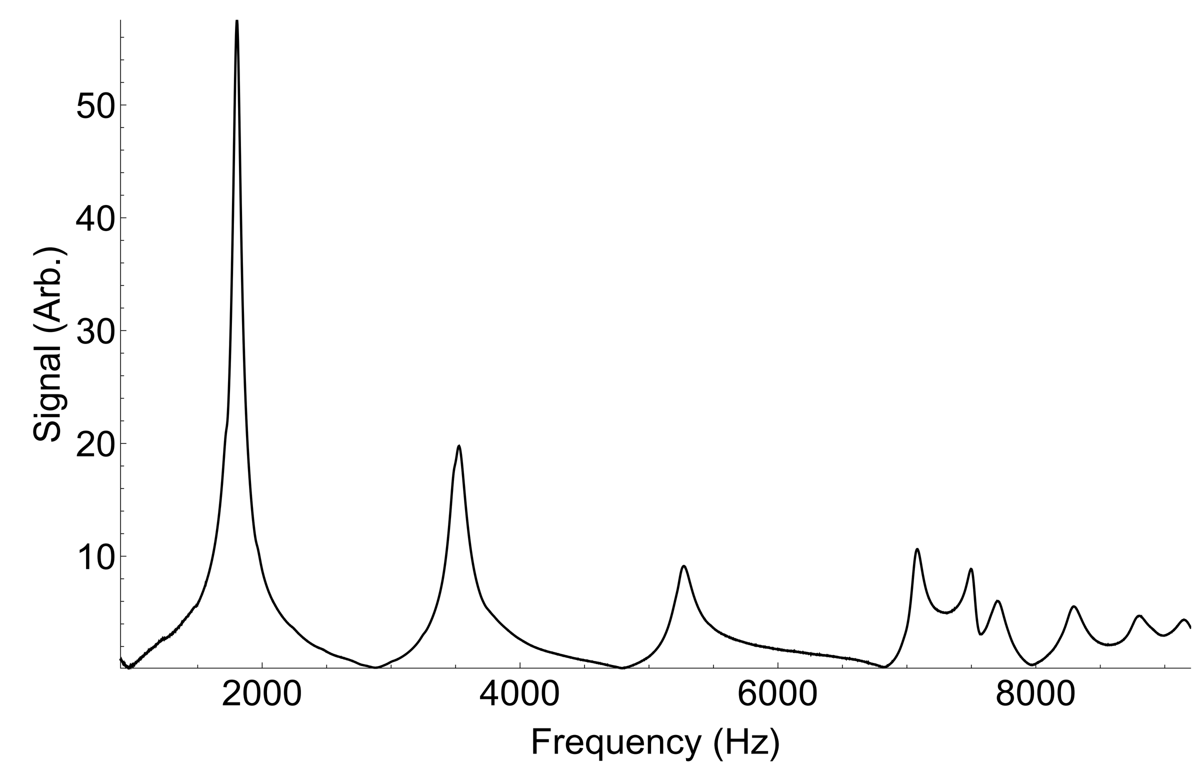

Figure 4: Microphone Signal vs. Frequency (Hz)

Table 1: Peak Fitting Data for fig. 4.

Peak Num	f0-Hz	Unc-F0	Gamma-Hz
1	1804.61	0.05	30.77
2	3517.07	0.15	55.61
3	5261.45	0.09	69.00

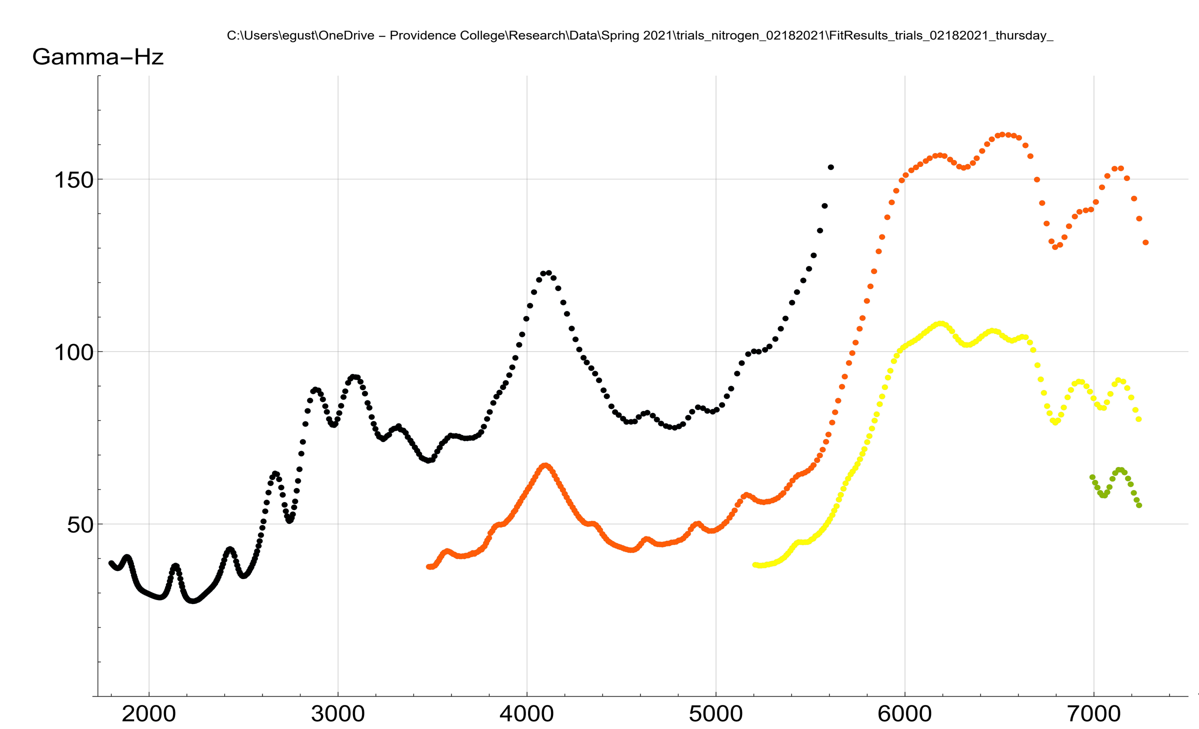

Figure 5: Resonance Peak Width (Hz) vs Frequency (Hz) for Nitrogen

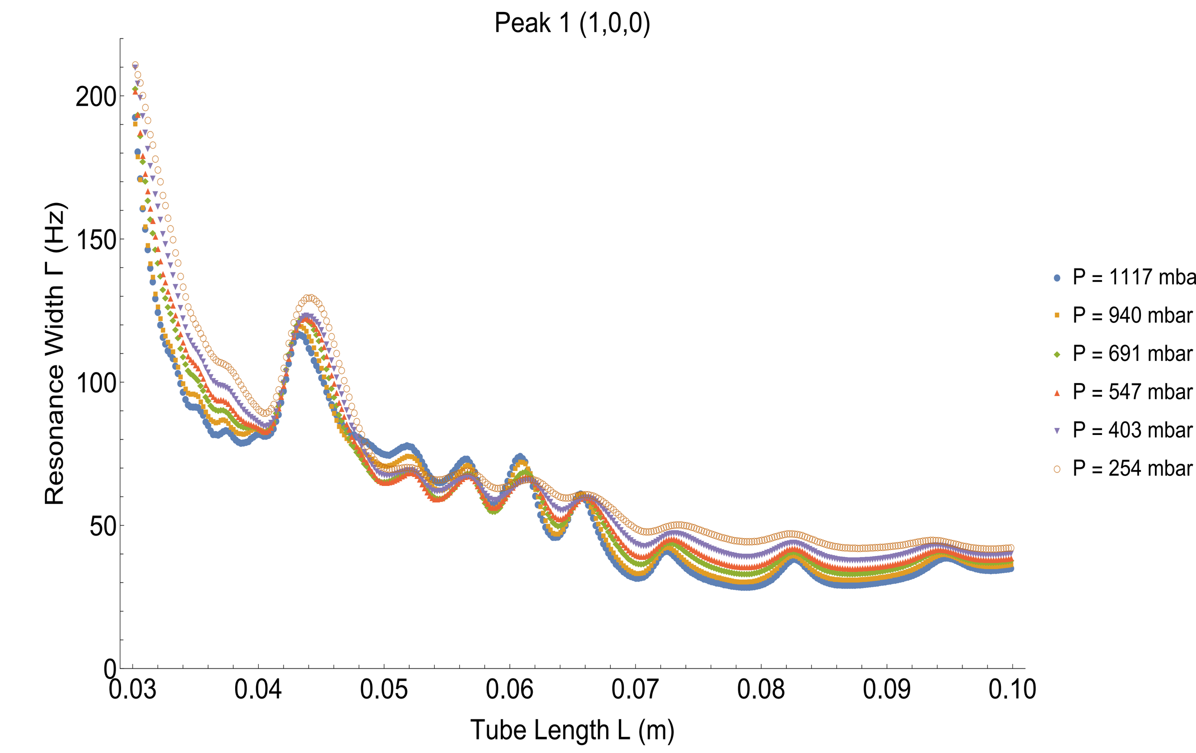

Figure 6: Resonance Width (Hz) vs. Tube Length (m) for one peak

Discussion

- In fig. 1, we can see a phase difference between the reference and microphone signals of about 0.3ms, and the period is about 0.5ms. From this figure we can see that a resonant frequency is close to 2000Hz. A better understanding of why the resonance does and does not occur at points would require comparison of more of these figures for a wider range of frequencies.
- In fig. 4, there is a noticeable pattern of the first three peaks being evenly spaced with their resonant frequencies being multiples of the leftmost peak. There is also a definite width to each of the first three peaks, with the tallest peak having the narrowest width. Above about 7000Hz, we have a more complicated pattern of resonances that are not evenly spaced.
- In table 1, the frequencies from F0-Hz are all roughly multiples of peak one. The first peak width is the narrowest of the three peaks, which agrees with the results seen in fig. 4. The pattern seen in fig.4 must then reflect the data itself having a pattern, not just the way the data was arranged in the graph. Unc-Gamma shows that there is a reasonably accurate uncertainty (<0.1%) for the frequencies themselves.
- In fig. 5, the widths of various peaks range from 20 – 150Hz. There is some consistency to this data. For instance, at any given resonance around 4000Hz, there is a larger width than at 4500Hz.
- Fig. 6 includes data from one peak at various tube lengths and pressures. While it might seem as if the varying of pressure has an impact on the resonant width, it is difficult to make a clear conclusion from fig. 6 because of how the graph is set up.

Conclusion

From this research, we obtained quantitative results from trials on various gases in a cylindrical cavity. The data itself is not able to show a direct trend between quantities such as variance of pressure because of of the many varying parameters. To come understand a relationship that might exist between pressure and the molecular behavior of gases, more trials with less varying parameters would be necessary. Continuing this research with a stronger focus on fewer varying parameters could reveal a clearer trend that exists with the behavior of gases.