

Application of Black-Scholes-Merton Model in Option Pricing and Intangibles Assets

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A patent is a document which describes an invention. Holding a patent gives the owner the right to develop the invention legally. For example, a patent of iOS allows Apple to manufacture products using the technology and similar accessories using the platform. The owner of a patent has the right to decide whether or not to publish the invention and start to generate income from its business. After investing an initial cost of developing the invention, the owner can run the business to receive income. If not, the maximum loss is the initial investment and the patent filing cost. This characteristic is similar to a call option which allows the holder to buy shares of stock at the strike price. The project attempt to price Apple's AirPods patent.

The more suitable model to price an intangible asset with potential economic benefit is the closely related Black-Scholes-Merton (BSM) model. It was first applied to pricing intangible assets by new modified model is first introduced by Dr. Aswath Damodaran of NYU.

$$C(S_t, t) = S_t e^{-yT} N(d_1) - e^{-rT} K N(d_2)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + (r + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}$$
$$d_2 = d_1 - \sigma\sqrt{T - t}$$

S: the current price of the stock/security.

K: the strike or exercise price of the option.

r: annual risk-free interest rate

T: time till time of expiration

σ : Volatility of the underlying asset; for example, the standard deviation of a stock's annual returns

N: standard normal cumulative distribution function

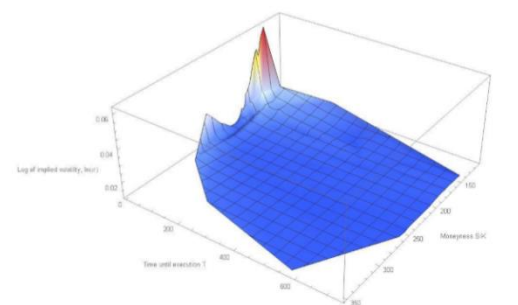
From caculation, the weighted average cost of capital of Apple Inc is 8.987%, which is used to be discount rate.

According to the assumptions of Black-Scholes model, volatility of the stock is known and constant throughout the lifetime of the option. Since Apple stock is the representative of the company's market value, it is realistic to use an estimate of the 10-year historical volatility of Apple's stock (Ticker: AAPL) in the calculation of a patent owned by Apple. Using AAPL stock closing prices available online (e.g. finance.yahoo.com) we determined the annualized volatility came out to $\sigma = 26.22\%$

	2016	2017	2018	2019	2020	2021	2022	2023	2035	2036
n	20	19	18	17	16	15	14	13	1	0
Sale (unit)		16	35	55	80	110	120	72	0.16	0.09
Sale (\$)		\$2,560	\$5,600	\$8,800	\$12,800	\$17,600	\$19,200	\$11,520	\$25.1	\$15
S	\$59,864	\$59,864	\$62,684	\$62,717	\$59,554	\$52,106	\$39,189	\$23,511	\$352	\$137
r		2.21%								
COGS		\$16,640	\$36,400	\$57,200	\$83,200	\$114,400	\$124,800	\$74,880	\$163	\$98
K	\$368,250	\$368,250	\$389,026	\$392,153	\$374,797	\$329,678	\$248,774	\$149,251	\$229	\$89
Profit (\$)		\$16,640	\$56,000	\$88,000	\$128,000	\$176,000	\$192,000	\$115,200	\$251	\$150

Forecasting Airpod's income streamline (in million)

The Black-Scholes-Merton model provides the estimated valuation of a patent accommodating the price movement with the effect of project delay on the price. It can also be used to evaluate the price of other intangible assets such as goodwill, brand name or copyrights. The idea is that based on the characteristic of the asset, we can forecast the future cash flow and estimate the net present value of the asset by a discounted cash flow model. For such cases, we can use the Black-Scholes-Merton model to compute the present value of the intangible asset as an option. However, the model also have some limits in estimating the price of options/assets. In the preceding example, the volatility is constant and known throughout the whole lifetime of the option. This assumption is definitely incorrect in real life. A company's volatility is always fluctuating and therefore never a constant. The concept of implied volatility, which arises in options pricing, can be used to produce a surface that usually contains a so-called "volatility smile" which indicates that the distribution of options with the same time to expiration will have a shape of a smile. In most cases, the distribution is skewed. Black-Scholes assumes that volatility is constant and remains so throughout the lifetime of the option. This in-turn would produce a volatility surface that is flat over time.



Apple's Implied Volatility surface derived from Apple's 0-180-day options

