

Gerrymandering and How to Find It

A Mathematical Approach

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Origins of Gerrymandering

- In 1812, Massachusetts Governor Elbridge Gerry approved a bill that would allow the state legislature to redraw district lines [2].
- New district boundaries resulted in disproportionate representation of the Democratic-Republican party over the Federalist party [2].
- One new district was shaped like a salamander, hence the name “Gerrymandering” [2].
- To this day, gerrymandering remains a problem, as it allows politicians to intentionally disenfranchise voters.



Figure 1: A rendition of the salamander-shaped district from the *Boston Gazette* in 1812.

Packing and Cracking

- Packing** is the process of forcing opposition voters into as few districts as possible [4].
- Cracking** is the process of splitting up opposition voters to dilute their voting power [4].

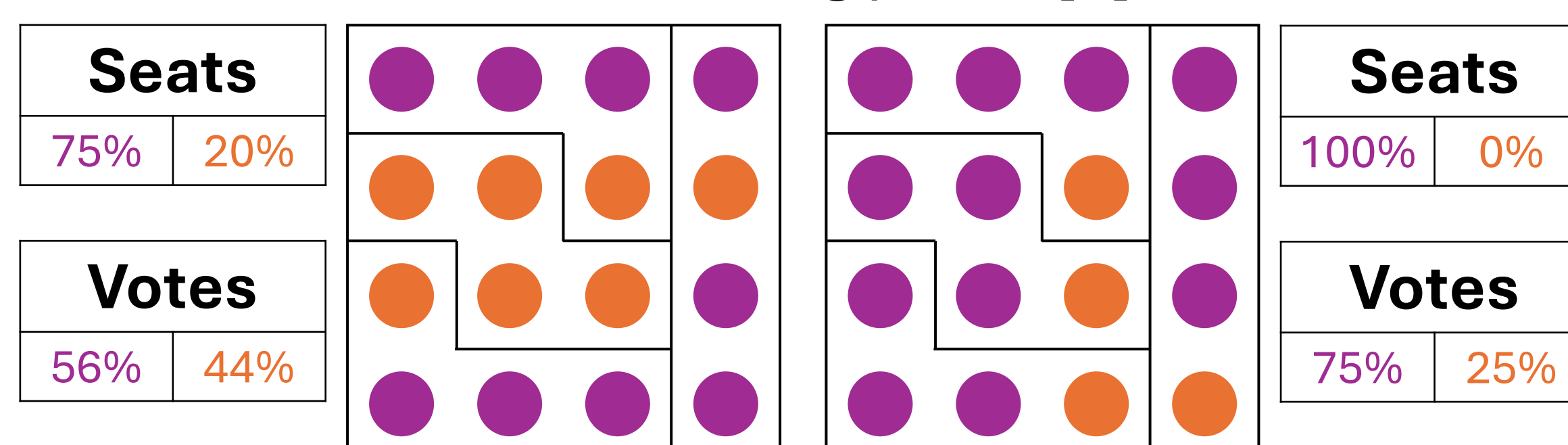


Figure 2: Examples of packing (left) and cracking (right). Note that both results lead to a disproportionate seat allocation.

- Most “effective” gerrymanders use a combination of packing and cracking to maximize seat gain for the party drawing the maps.

The Efficiency Gap

- Proposed in 2014 by Nicholas Stephanopoulos and Eric McGhee [3].
- Measures the amount of “wasted” votes per party.
- A positive score shows Republican bias, and a negative score shows Democratic bias [3].
- Formula [3]:

$$w_i = \begin{cases} 1 & \text{if the party lost district } i \\ 0 & \text{if the party won district } i \end{cases}$$
$$P_W = \sum_{i=1}^n \left[w_i \cdot \left(p_i - \frac{v_i}{2} \right) + (1 - w_i) \cdot p_i \right]$$
$$EG = \frac{D_W - R_W}{V}$$

- Example: 2022 Kansas
 - Efficiency Gap Score: **5.9%**
 - Slight Republican Bias

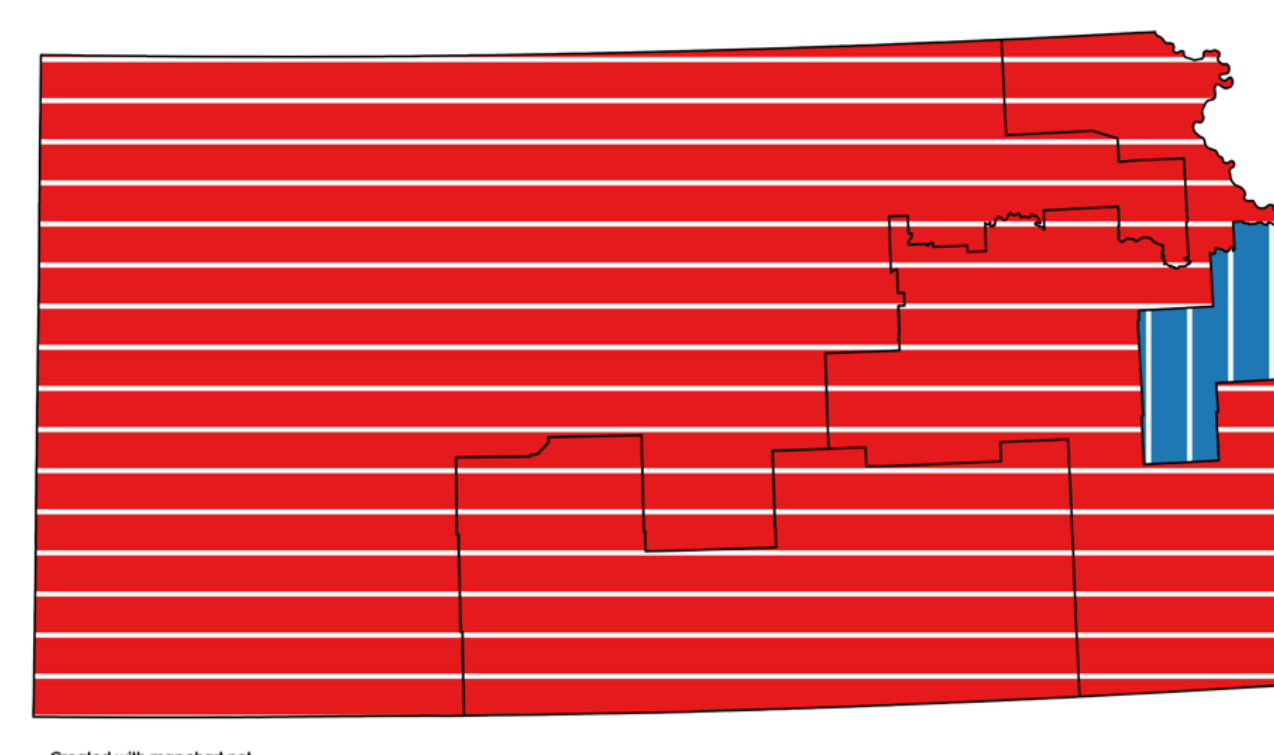


Figure 3: The 2022 results of the United States House of Representatives Election in Kansas. Vertical, blue stripes represent Democrat-won seats, and horizontal, red stripes represent Republican-won seats.

- The Efficiency Gap considers the effects of packing and cracking.
 - Concentrating opposition voters in as few districts as possible to minimize the impact of their vote.
 - Diluting opposition voters so that the party wins fewer seats.

The Polsby-Popper Score

- Measures compactness of a district, or how “normal” the district boundaries are [1].
- Compares the area of the district to the area of a circle with the same perimeter [1].
- Formula [1]:

$$PP(D) = \frac{4\pi \cdot A(D)}{P(D)^2} = \frac{A(D)}{\pi r^2}$$

- Benefits:
 - A district with a low score in the *interior* of a state may be a sign of gerrymandering.
 - The Polsby-Popper score provides a straightforward metric to measure unfair maps.
- Drawbacks:
 - The Polsby-Popper score does not directly measure gerrymandering.
 - Some states have borders that are naturally not compact, leading to artificially low scores.
 - The range from 0 to 1 of the score provides unattainable standards, since an exact score of 0 or 1 is statistically unlikely [1].

References

- [1] Moon Duchin and Olivia Walch. *Political Geometry: Rethinking Redistricting in the US with Math, Law, and Everything in Between*. Birkhäuser, 2022. ISBN: 9783319691602.
- [2] Elmer Cummings Griffith. *The Rise and Development of the Gerrymander*. Scott Fores-man and Company, 1907.
- [3] Nicholas O. Stephanopoulos and Eric M. McGhee. “Partisan Gerrymandering and the Efficiency Gap”. In: *The University of Chicago Law Review* 82.2 (2015), pp. 831–900. DOI: <https://www.jstor.org/stable/43410706>.
- [4] Kristopher Tapp. “Measuring Political Gerrymandering”. In: *The American Mathematical Monthly* 126.7 (2019), pp. 593–609. DOI: <https://www.jstor.org/stable/48662153>.